

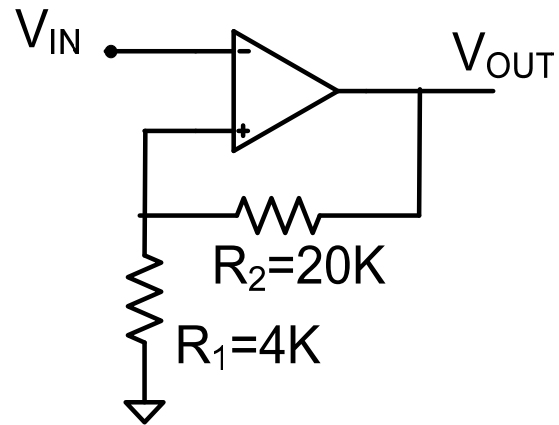
# EE 230

## Lecture 22

Nonlinear Op Amp Applications  
– Waveform Generators

# Quiz 16

Plot the transfer characteristics of the following circuit. Assume the op amp has  $V_{SATH}=12V$  and  $V_{SATL}=-12V$ .



And the number is ?

1

3

8

5

4

?

2

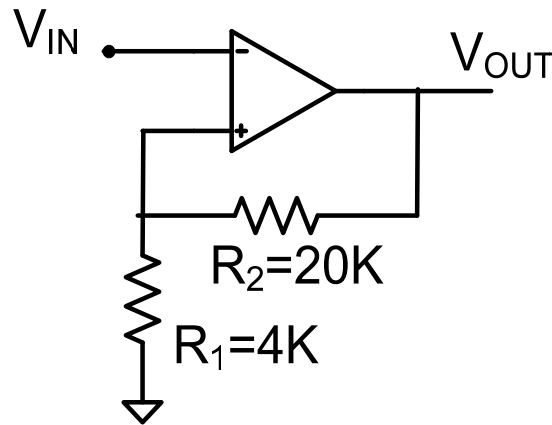
6

9

7

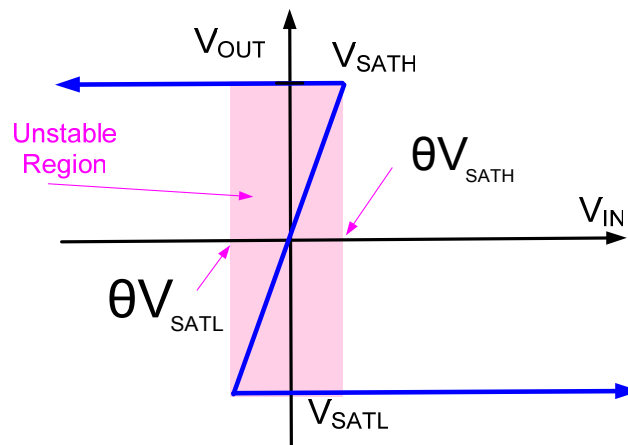
# Quiz 16

Plot the transfer characteristics of the following circuit. Assume the op amp has  $V_{SATH}=12V$  and  $V_{SATL}=-12V$ .



Solution

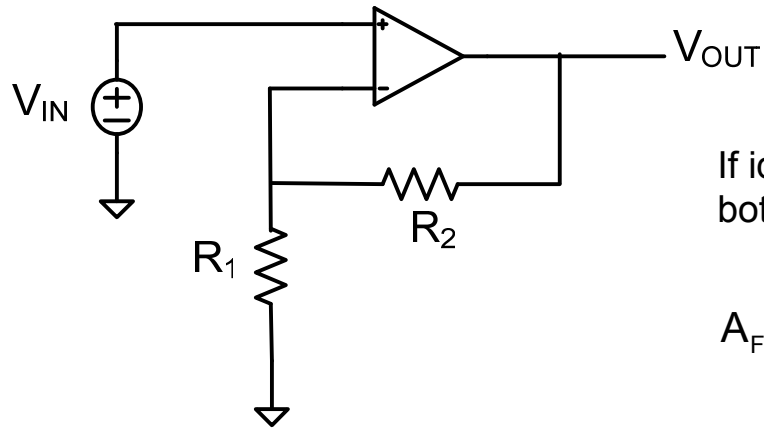
Define  $\theta = \frac{R_1}{R_1 + R_2}$



## Review from Last Lecture



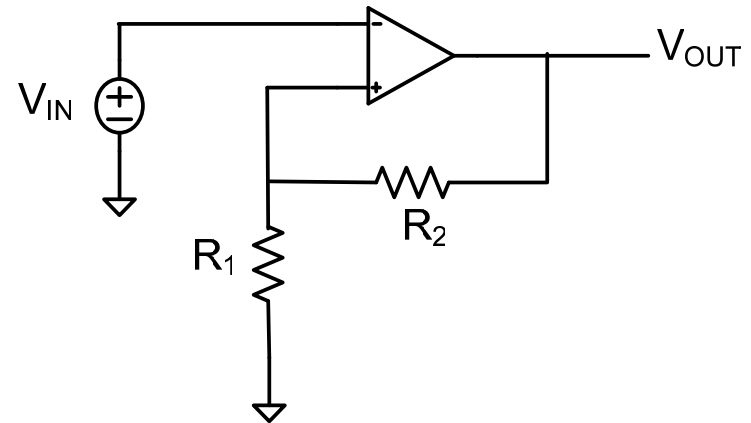
Before trashing the “bad” circuit which we saw was unstable, lets see if it has any useful properties!



If ideal op amps  
both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$

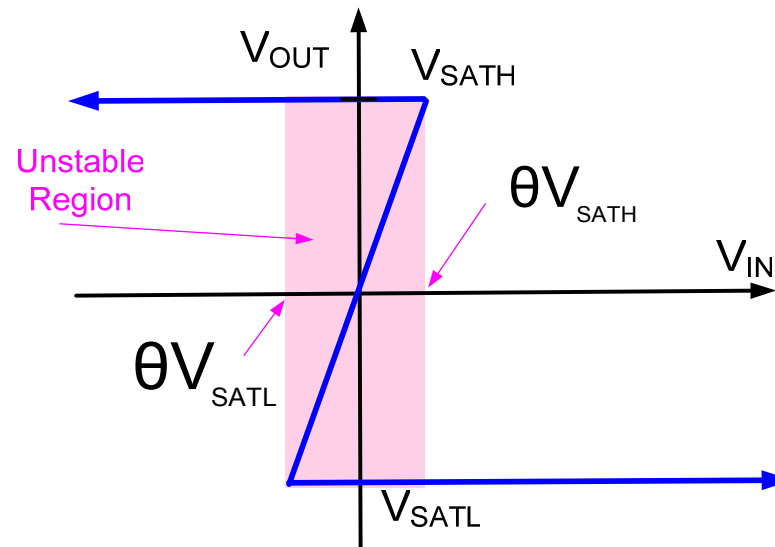
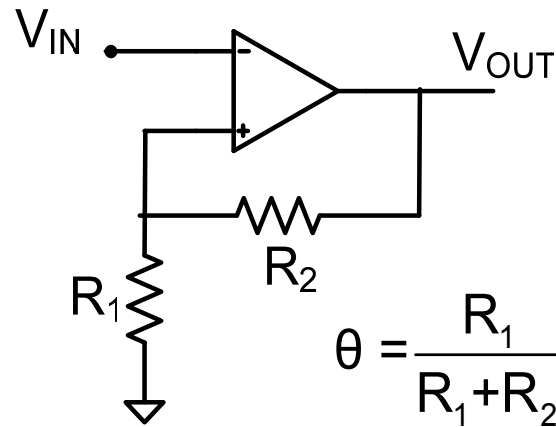
Usually the good circuit



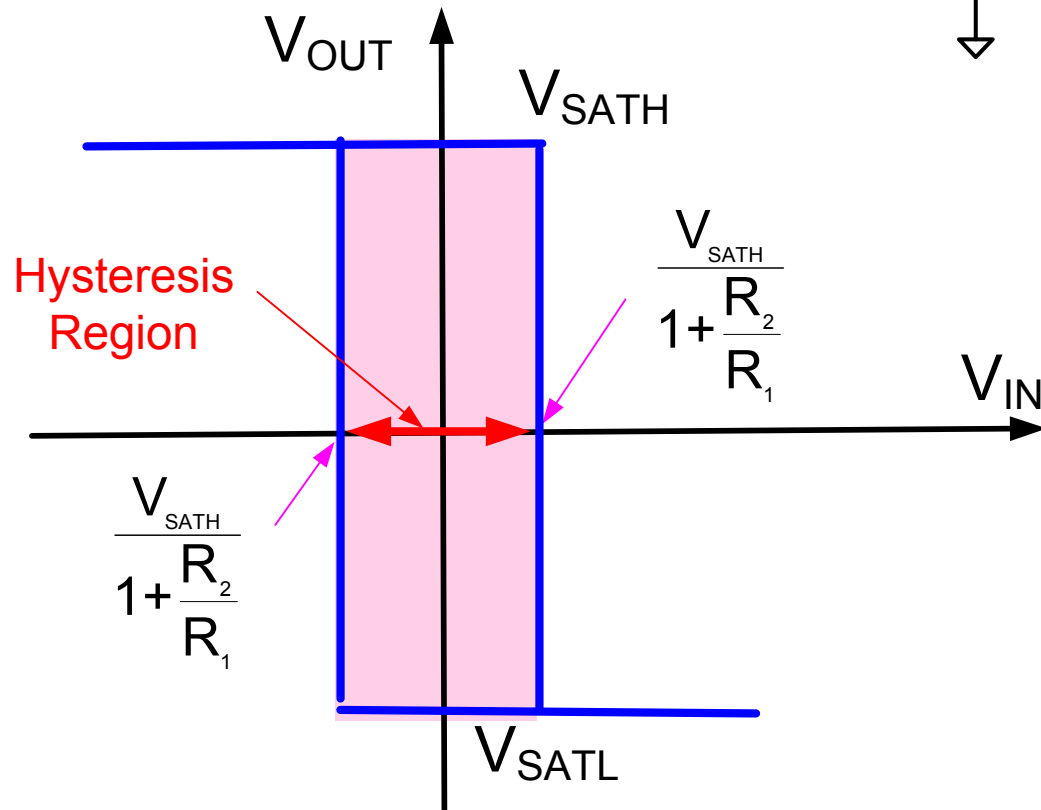
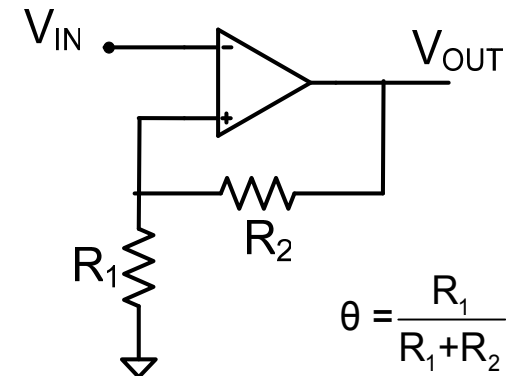
Usually the bad circuit

This circuit is unstable !

# Comparator with Hysteresis

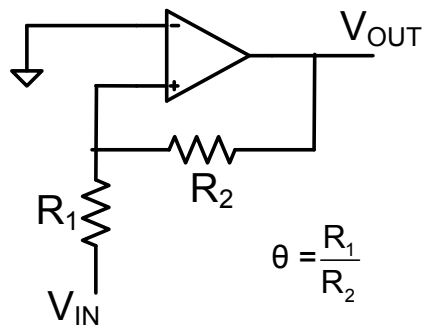
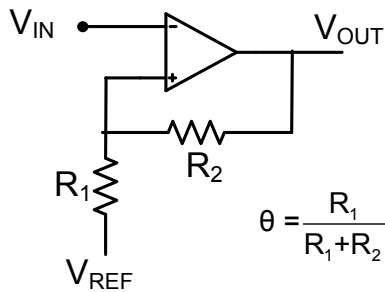
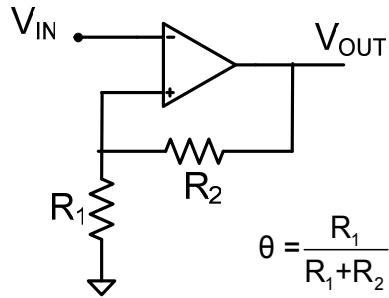


# Comparator with Hysteresis

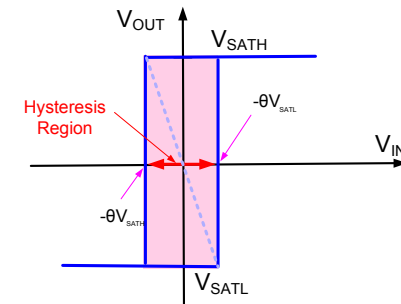
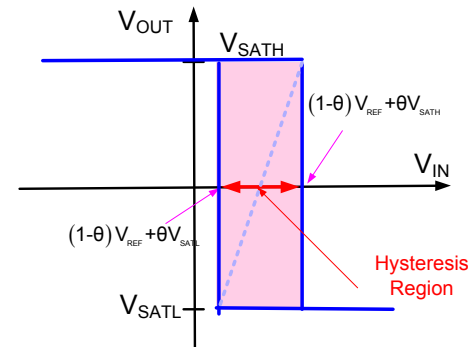
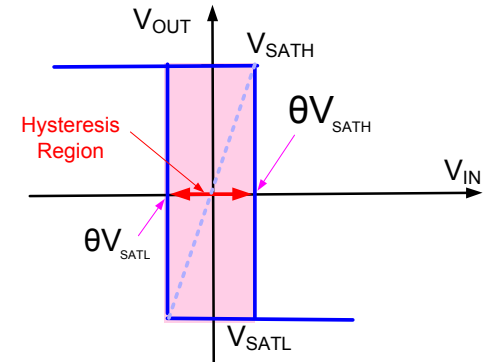


## Review from Last Lecture

# Modifications of Comparator with Hysteresis



Note this is the basic inverting amplifier with op amp terminals interchanged

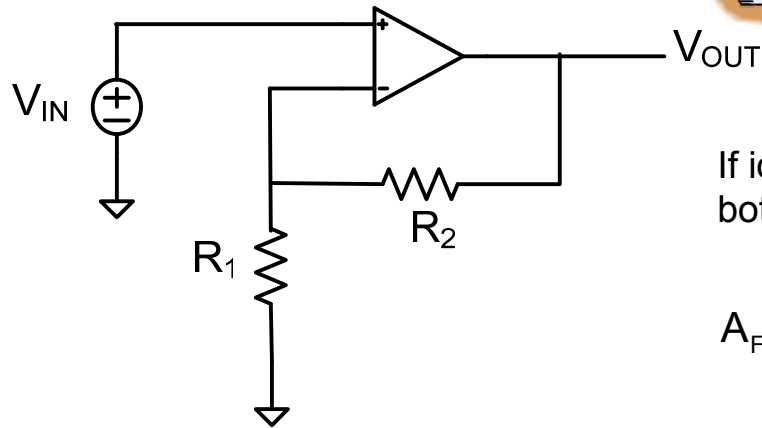


Many other ways to control position and size of hysteresis window





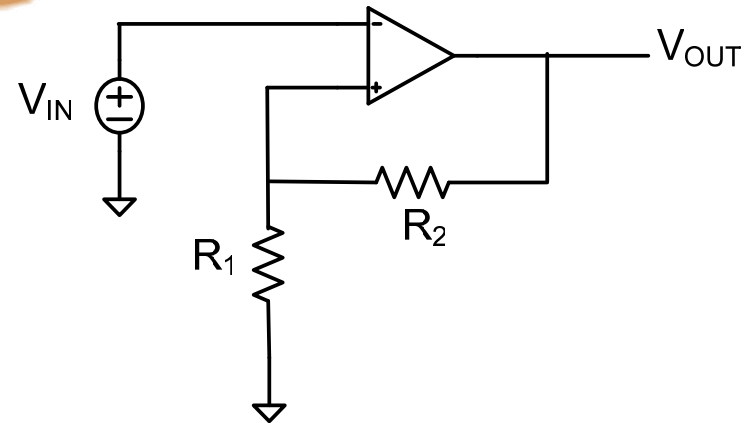
How about the other circuit?



Usually the good circuit

If ideal op amps  
both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$

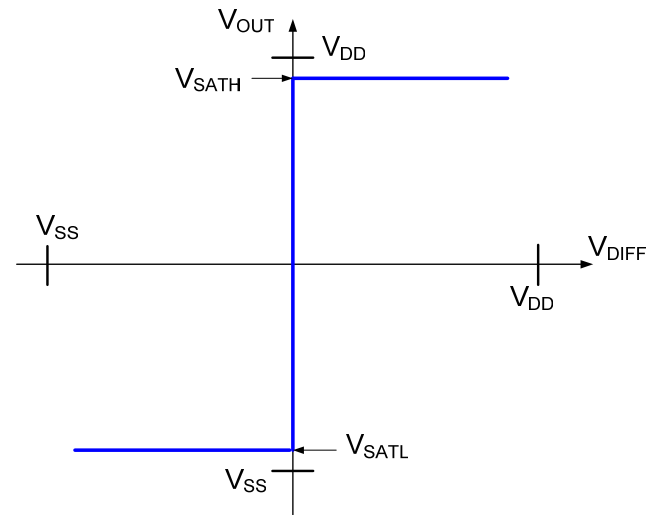
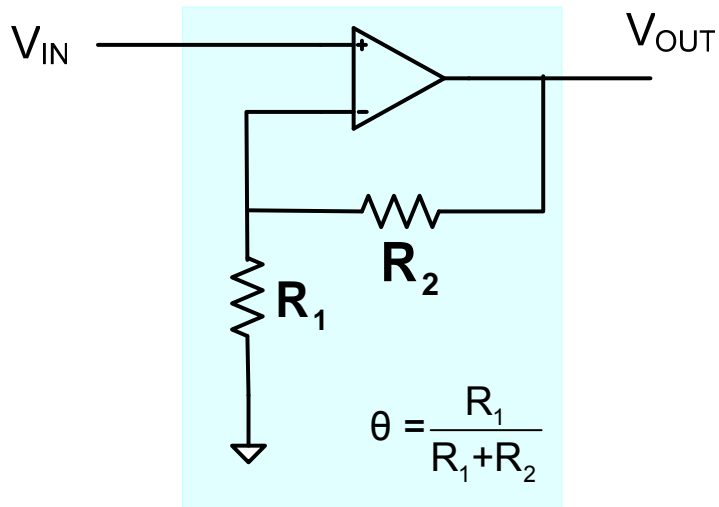


Usually the bad circuit

This circuit is unstable !

Will now analyze the “usually good” circuit using nonlinear analysis methods

# Consider this circuit



$$V_{OUT} = \begin{cases} V_{SATH} & V_{DIFF} > V_{SATH}/A_0 \\ A_0 V_{IN} & V_{SATL}/A_0 < V_{DIFF} < V_{SATH}/A_0 \\ V_{SATL} & V_{DIFF} < V_{SATL}/A_0 \end{cases}$$

Region 1  
Region 2  
Region 3

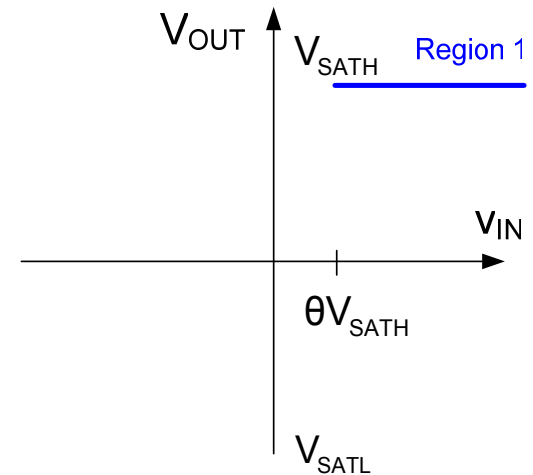
Assume in Region 1 (must verify)

$$V_{OUT} = V_{SATH}$$

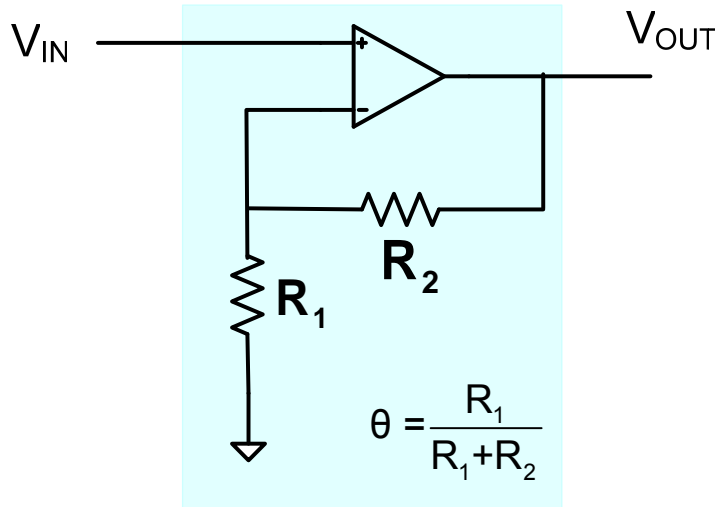
Valid for

$$V_{IN} - \theta V_{SATH} > \frac{V_{SATH}}{A_0}$$

$$\theta V_{SATH} < V_{IN}$$



# Consider this circuit



$$V_{OUT} = \begin{cases} V_{SATL} \\ A_0 V_{IN} \\ V_{SATH} \end{cases}$$

$$\begin{cases} V_{DIFF} > \frac{V_{SATH}}{A_0} \\ \frac{V_{SATL}}{A_0} < V_{DIFF} < \frac{V_{SATH}}{A_0} \\ V_{DIFF} < \frac{V_{SATL}}{A_0} \end{cases}$$

Region 1

Region 2

Region 3

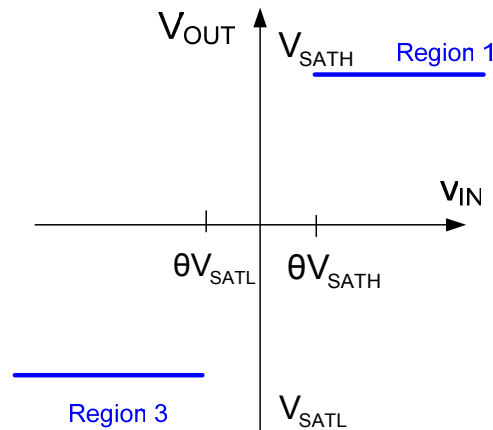
Assume in Region 3 (must verify)

$$V_{OUT} = V_{SATL}$$

Valid for

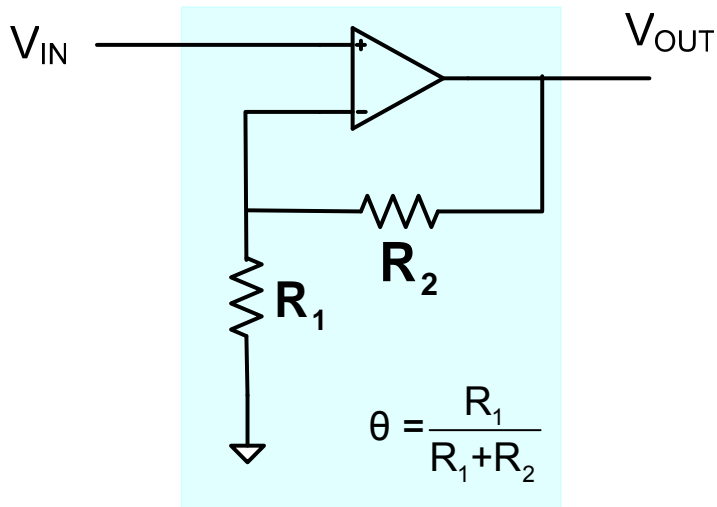
$$V_{IN} - \theta V_{OUT} < \frac{V_{SATL}}{A_0}$$

$$\theta V_{SATL} > V_{IN}$$



Note this is now single-valued for the range considered so far

# Consider this circuit



$$V_{OUT} = \begin{cases} V_{SATL} & V_{DIFF} < \frac{V_{SATL}}{A_0} \\ A_0 V_{IN} & \frac{V_{SATL}}{A_0} < V_{DIFF} < \frac{V_{SATH}}{A_0} \\ V_{SATH} & V_{DIFF} > \frac{V_{SATH}}{A_0} \end{cases}$$

$$\begin{cases} V_{DIFF} > \frac{V_{SATH}}{A_0} & \text{Region 1} \\ \frac{V_{SATL}}{A_0} < V_{DIFF} < \frac{V_{SATH}}{A_0} & \text{Region 2} \\ V_{DIFF} < \frac{V_{SATL}}{A_0} & \text{Region 3} \end{cases}$$

Region 1  
Region 2  
Region 3

Assume in Region 2 (must verify)

$$V_{DIFF} = V_{IN} - \theta V_{OUT}$$

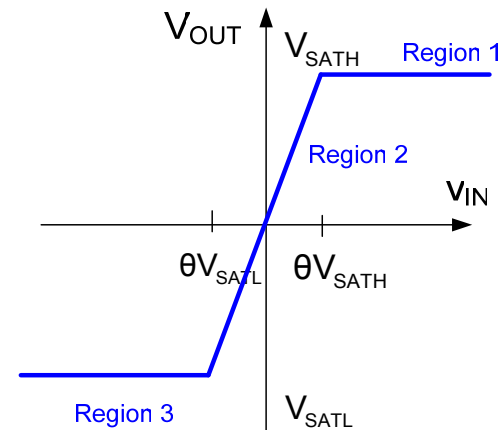
$$V_{OUT} = A_0 V_{DIFF}$$

Valid for

$$\frac{V_{SATL}}{A_0} < V_{IN} - \theta V_{OUT} < \frac{V_{SATH}}{A_0}$$

$$V_{OUT} = \left( \frac{1}{\theta + A_0^{-1}} \right) V_{IN}$$

$$V_{OUT} = \left( \frac{1}{\theta} \right) V_{IN}$$

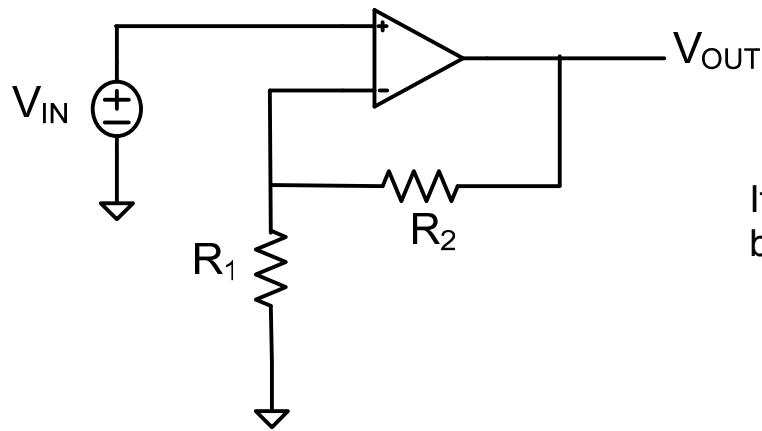


(must use exact value for  $V_{OUT}$  to avoid degenerate conditions)

$$\theta V_{SATL} < V_{IN} < \theta V_{SATH}$$

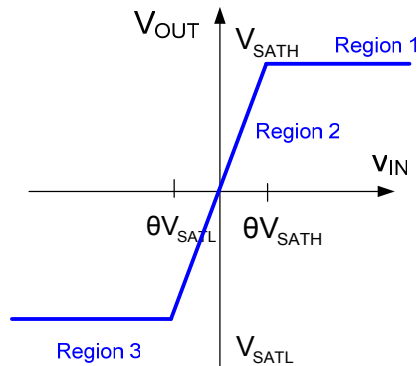
- Note this circuit does not have a hysteresis loop
- Simply serves as a noninverting amplifier that saturates at extreme inputs

# Comparison of basic noninverting amplifier structures

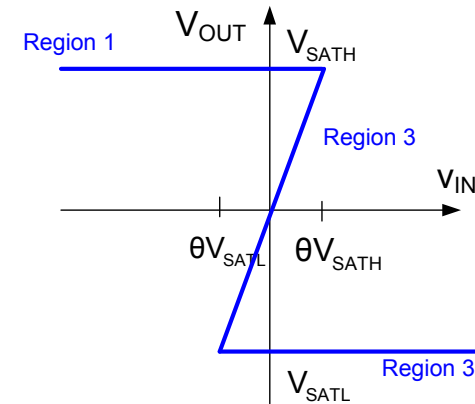
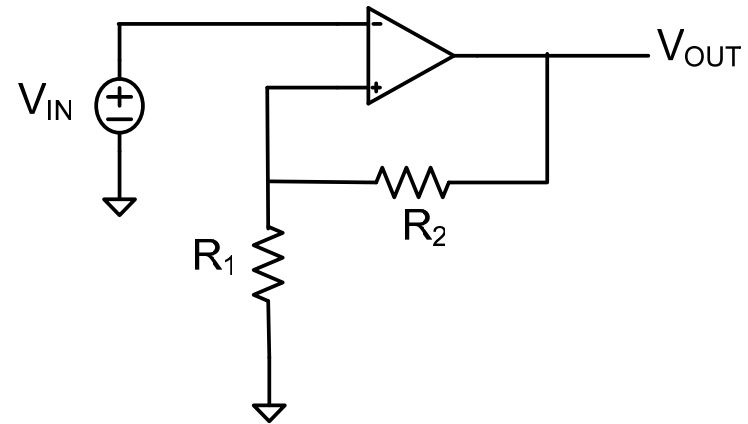


If ideal op amps  
both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$

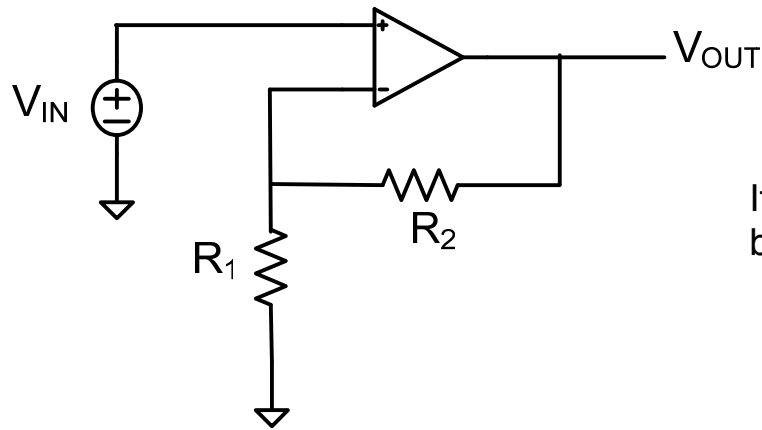


- Serves as an amplifier directly
- Stable
- No hysteresis loop

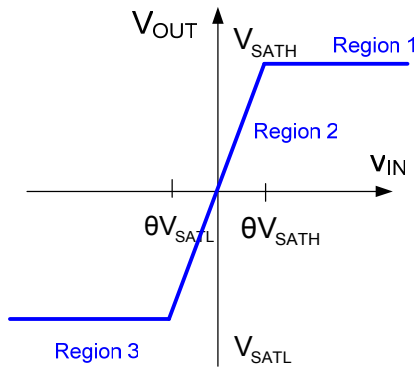


- Not useful as an amplifier directly
- Unstable
- Serves as comparator with hysteresis

# Comparison of basic noninverting amplifier structures

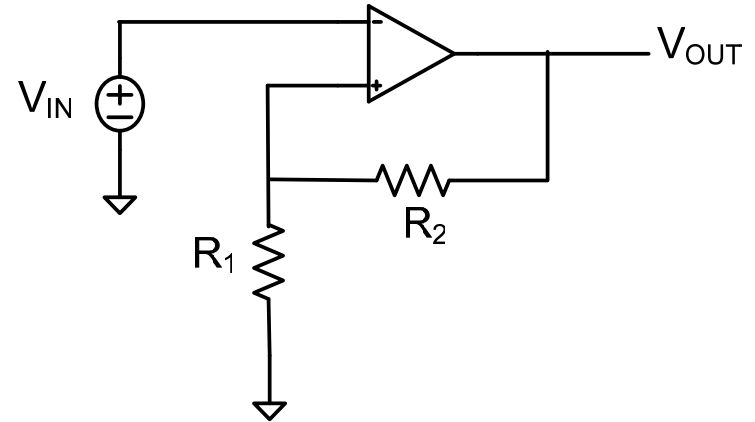


Usually a good amplifier

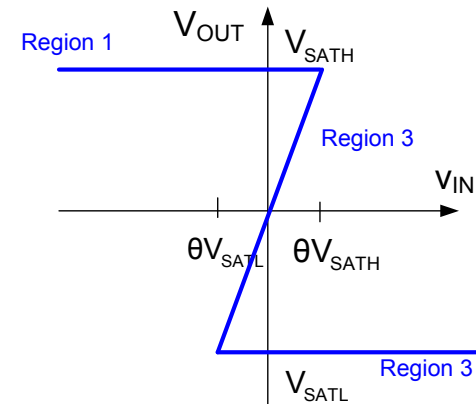


If ideal op amps both have gain

$$A_{FB} = 1 + \frac{R_2}{R_1}$$

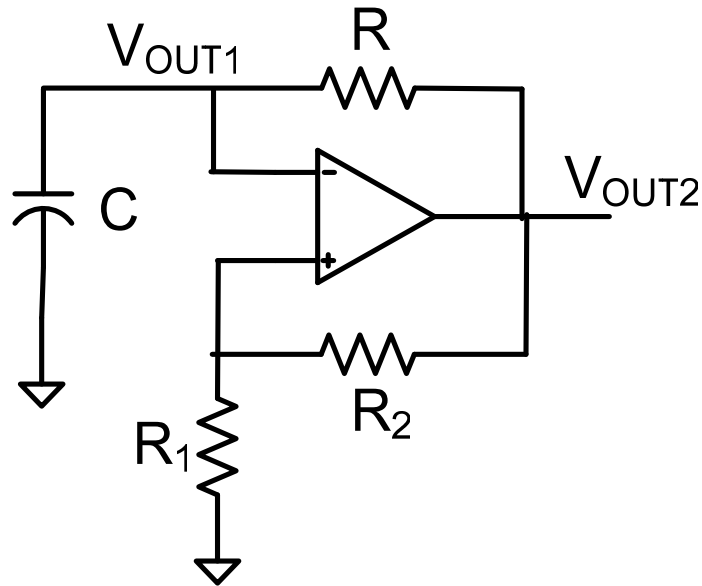


Usually not a good amplifier



Give examples where the circuit the left serves as an amplifier and that on the right does not

# Waveform Generator



$$\theta = \frac{R_1}{R_1 + R_2}$$

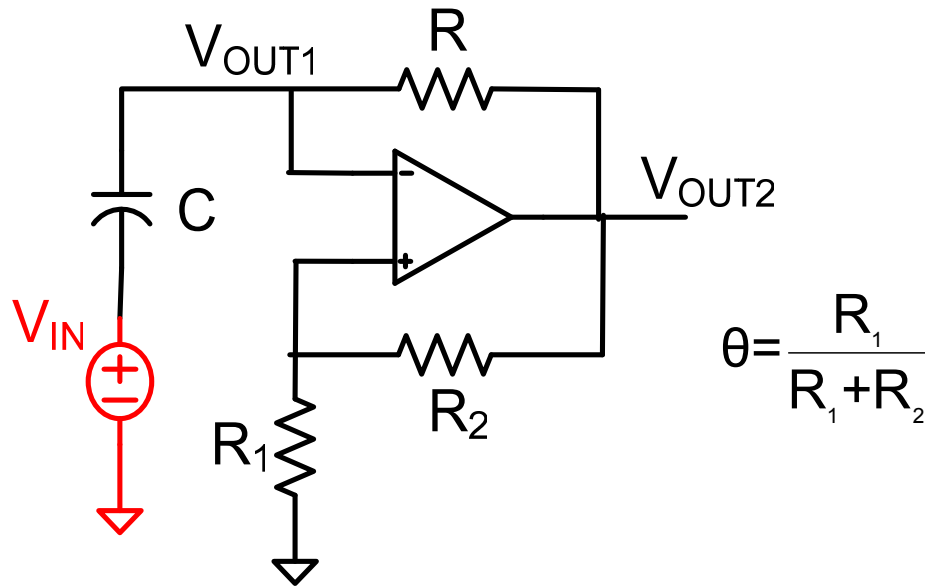
## Lets first check stability

Will add an excitation that does not change the “dead” network to obtain a transfer function

The roots of the denominator are the poles of the circuit

Actually can check stability without adding input but will not go into details at this time

# Waveform Generator



Lets first check stability

$$V_{OUT2} = \frac{GB}{s} (\theta V_{OUT2} - V_{OUT1})$$

$$V_{OUT1} (sC + G) = GV_{OUT2} + sCV_{IN}$$

$$T(s) = \frac{V_{OUT2}}{V_{IN}} = \frac{sC}{s^2 \left( \frac{C}{GB} \right) + s \left( \frac{G}{GB} - \theta C \right) + G(1-\theta)}$$

$$T(s) \approx \frac{sC}{s^2 \left( \frac{C}{GB} \right) + s(-\theta C) + G(1-\theta)}$$

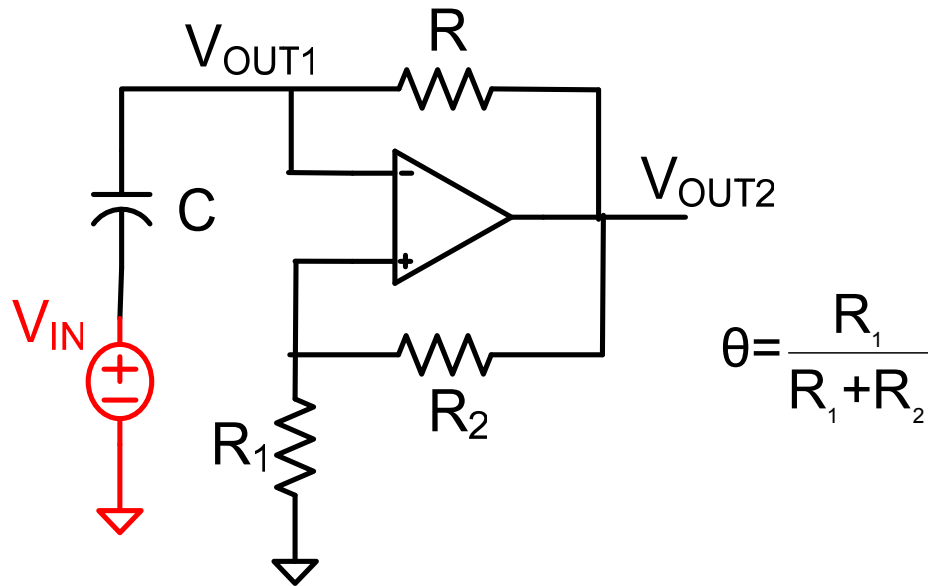
It follows that there is a pole on the positive real axis !

Thus the circuit is unstable and will operate nonlinearly !

Can find the RHP pole of THIS circuit even if op amp is ideal!



# Waveform Generator



Can also check stability for this with ideal op amp

$$V_{OUT1} = \theta V_{OUT2}$$

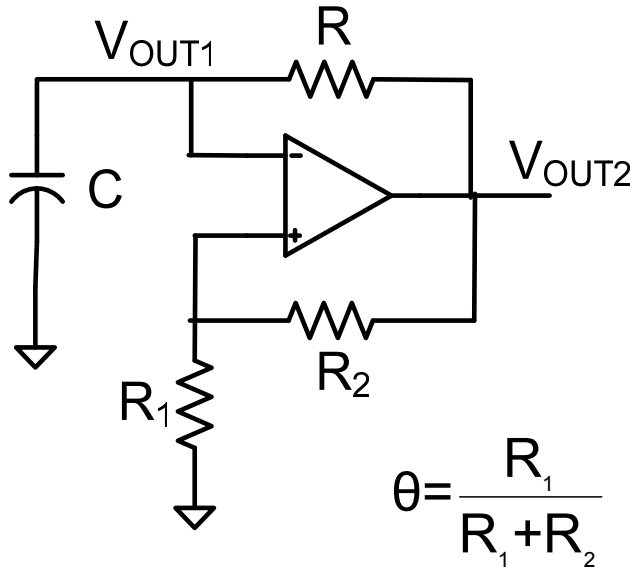
$$V_{OUT1}(sC + G) = G V_{OUT2} + sC V_{IN}$$

$$T(s) \approx \frac{sC}{s(-\theta C) + G(1-\theta)}$$

Pole at  $p = \frac{G(1-\theta)}{\theta C}$

Observe the pole is on the positive real axis !  
Thus the circuit is unstable and will operate nonlinearly !

# Waveform Generator



$$\theta = \frac{R_1}{R_1 + R_2}$$

Assume  $V_{OUT2}$  is in either the high or low state at any time  $t$

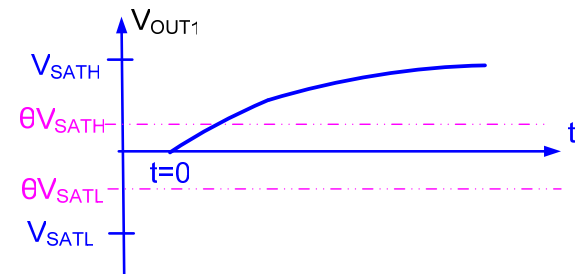
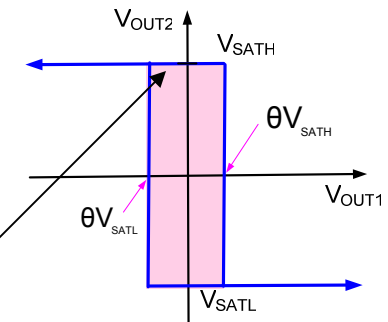
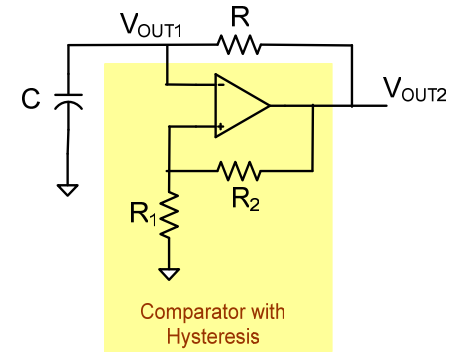
w.l.o.g. assume that  $V_{OUT1} = 0$  at  $t=0$  and  $V_{OUT2} = V_{SATH}$

$$V_{OUT1} = F + (I - F)e^{-\frac{t}{RC}} \quad F = V_{SATH}, \quad I = 0$$

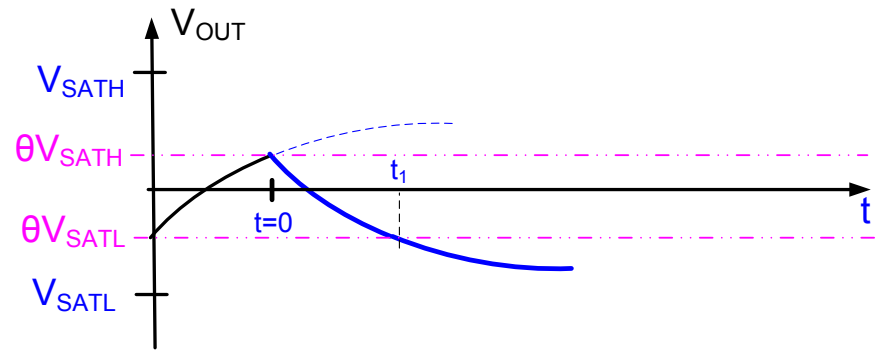
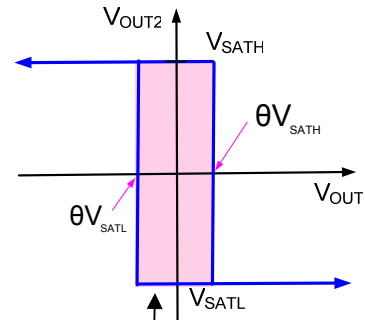
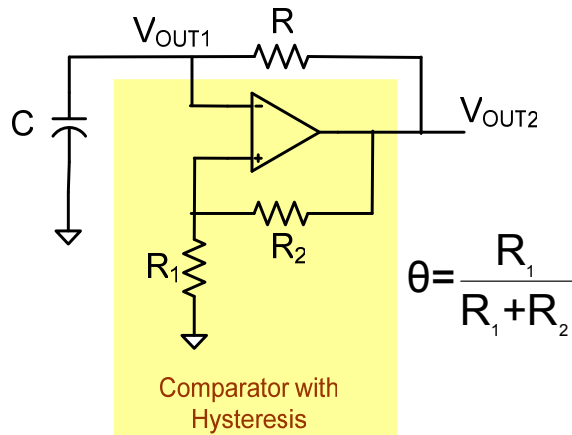
$$V_{OUT1} = V_{SATH} \left( 1 - e^{-\frac{t}{RC}} \right)$$

while in this state,  $V_{OUT1}$  is increasing, however, this is valid only for  $V_{OUT1} < \theta V_{SATH}$

As soon as  $V_{OUT1} = \theta V_{SATH}$ , comparator output will switch to  $V_{SATL}$



# Waveform Generator



For convenience reset the time axis as shown  
now, for  $t > 0$  have

$$V_{OUT1} = F + (I - F)e^{-\frac{t}{RC}} \quad F = V_{SATL}, I = \theta V_{SATH}$$

$$V_{OUT1} = V_{SATL} \left( 1 - e^{-\frac{t}{RC}} \right) + \theta V_{SATH} e^{-\frac{t}{RC}}$$

while in this state,  $V_{OUT1}$  is decreasing, however, this is valid only for  $V_{OUT1} > \theta V_{SATL}$

As soon as  $V_{OUT1} = \theta V_{SATL}$ , comparator output will switch to  $V_{SATH}$

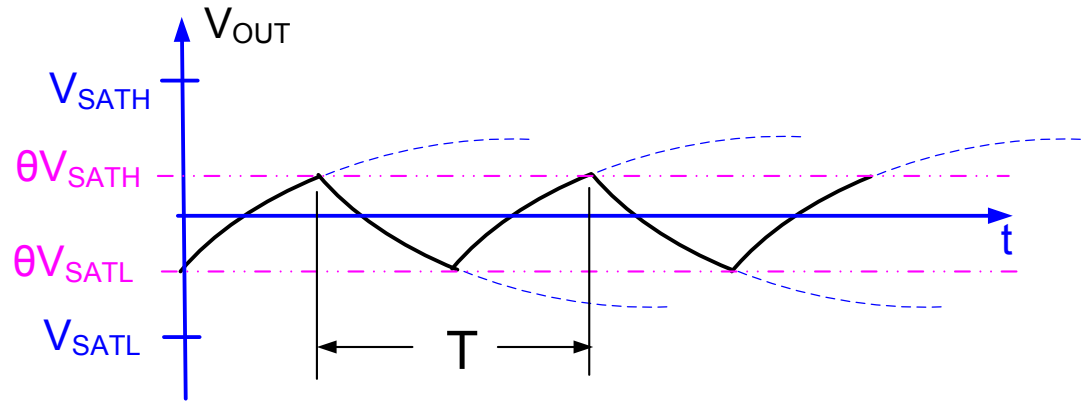
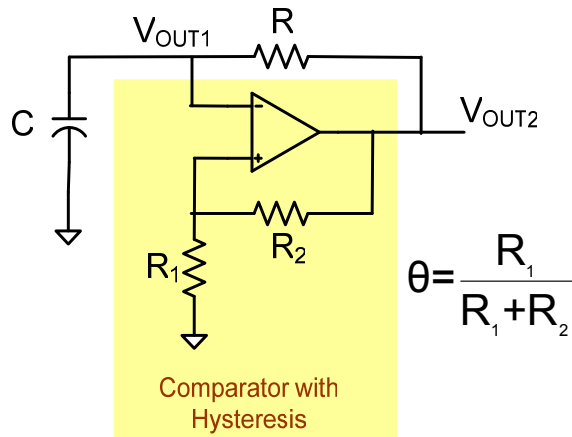
If we define  $t_1$  to be the time where this switch occurs, it follows that

$$\theta V_{SATL} = V_{SATL} \left( 1 - e^{-\frac{t_1}{RC}} \right) + \theta V_{SATH} e^{-\frac{t_1}{RC}}$$

Solving for  $t_1$ , obtain

$$t_1 = -RC \ln \left( \frac{V_{SATL} (\theta - 1)}{\theta V_{SATH} - V_{SATL}} \right)$$

# Waveform Generator



$$t_1 = -RC \ln \left( \frac{V_{SATL} (\theta - 1)}{\theta V_{SATH} - V_{SATL}} \right)$$

this process repeats itself

the rise time and the fall times are identical

the period of the nearly triangular waveform is thus  $2t_1$

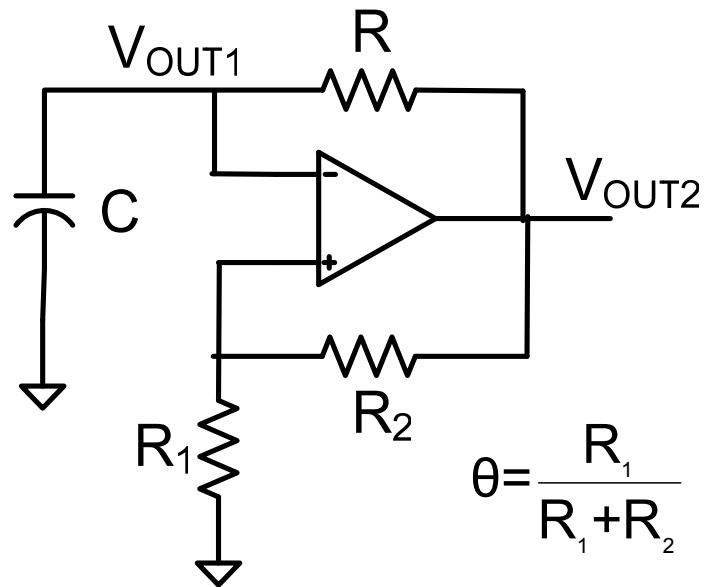
$$T = 2t_1 = -2RC \ln \left( \frac{V_{SATL} (\theta - 1)}{\theta V_{SATH} - V_{SATL}} \right)$$

If  $V_{SATL} = -V_{SATH}$ , this simplifies to

$$f = \frac{1}{T} = \frac{1}{2RC} \frac{1}{\ln \left( \frac{\theta V_{SATH} - V_{SATL}}{V_{SATL} (\theta - 1)} \right)}$$

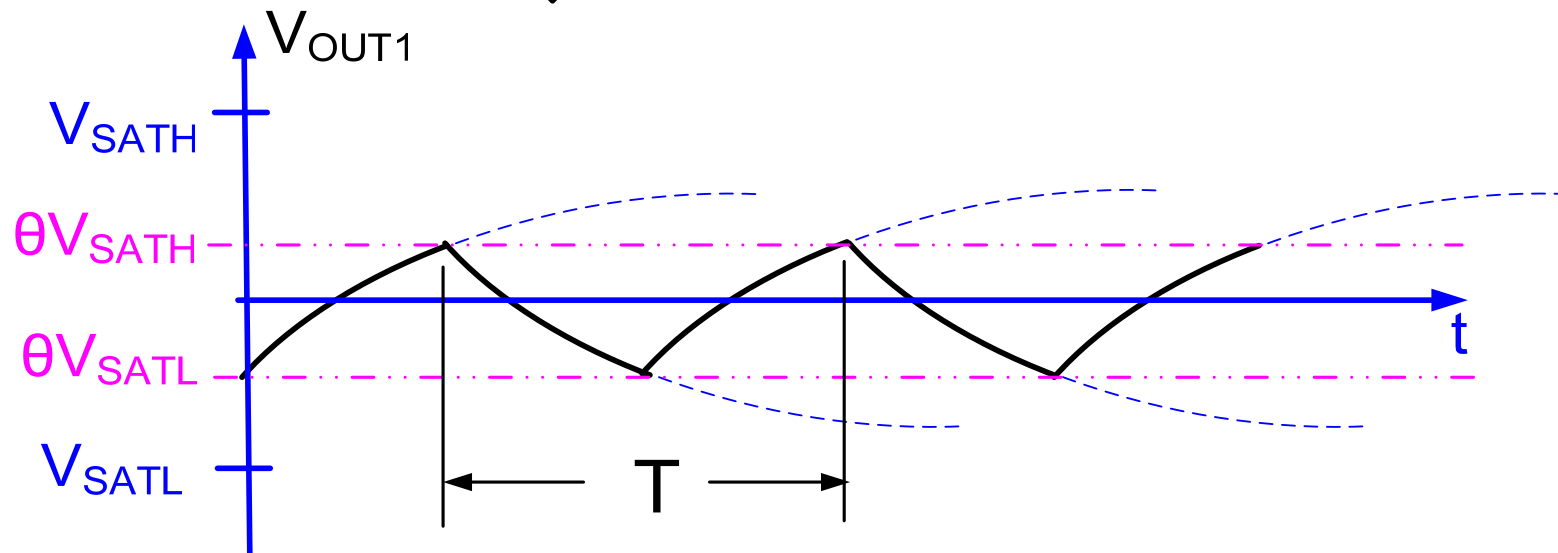
$$f = \frac{1}{2RC} \frac{1}{\ln \left( \frac{1 + \theta}{1 - \theta} \right)}$$

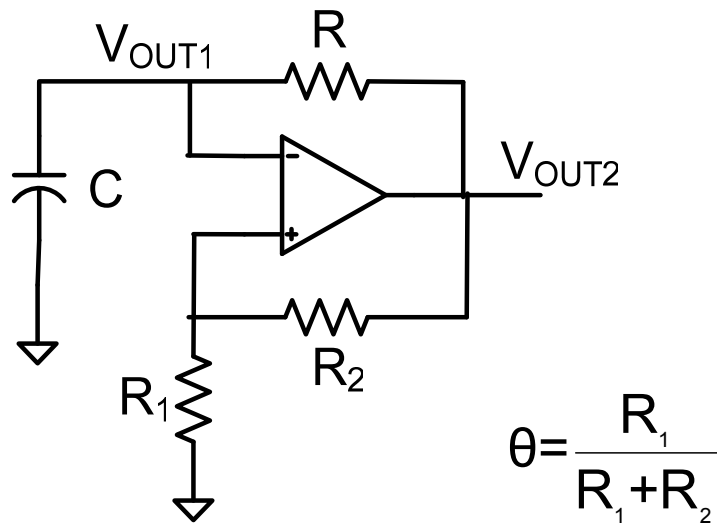
## Waveform Generator



for  $V_{SATL} = -V_{SATH}$

$$T = 2RC \ln \left( \frac{1 + \theta}{1 - \theta} \right)$$

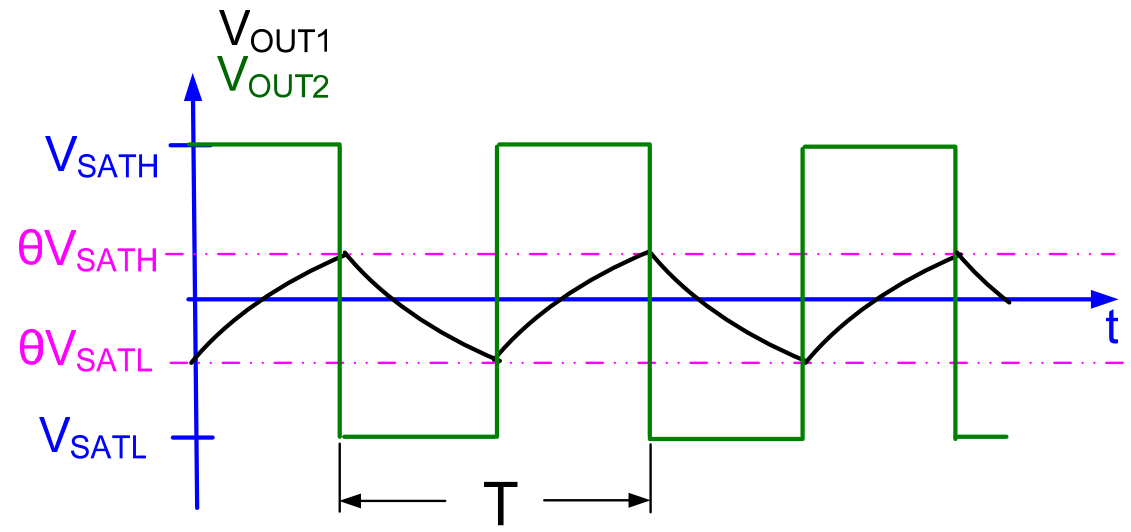




$$\theta = \frac{R_1}{R_1 + R_2}$$

for  $V_{SATL} = -V_{SATH}$

$$f = \frac{1}{2RC} \frac{1}{\ln\left(\frac{1+\theta}{1-\theta}\right)}$$



Square and distorted triangular output waveforms  
 Slope of square wave is determined by SR of Op Amp

**End of Lecture 22**